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A STUDY ON OPTIMUM INVENTORY CONTROL SUPPLY CHAIN POLICY WITH VARYING COST

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Abstract. In this paper we develop a supply chain inventory policy with varying transportation cost and unit price. We formulate a mathematical model using various costs involved in the inventory to determine the Economic ordering quantity to be purchased by the decision maker and optimum cost to the business for running a smooth supply chain system. Using the differential equation, we minimize the total inventory cost and quantity which would be effective for the business and optimize the inventory levels, reduce the holding cost, balancing stock outs and improve the overall supply chain system. The numerical examples have been illustrated.
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1. Introduction

Supply chain management and inventory control are essential for making sure that firms, especially those in the manufacturing, distribution, and retail industries, run smoothly. The process of observing and controlling the movement of items from the stock room is known as inventory control. It involves controlling the stock levels to make sure there is never an overabundance or shortage of goods. The foremost goals of inventory control are to reduce holding costs and avoid other costs, to balance inventory levels to meet customer demand without overstocking or under stocking, and to simplify inventory management procedures like ordering, receiving, storing, and tracking. Coordination and integration of multiple operations both inside and outside of businesses are key components of supply chain management, which guarantees the smooth relocation of goods and services from the point of source to the final consumer. It includes every step of the production and distribution process, from locating items to the last delivery.

The procurement of finished goods or raw materials from suppliers, product assembly or manufacturing, storage, management, and transportation to distribution centres or directly to customers are among the essential elements of supply chain management. Other essential elements include controlling inventory, warehousing, and transportation within the supply chain and forecasting customer demand to align production and inventory levels. Therefore, inventory control is the precise process of controlling the amount of inventory that a business has on hand, whereas supply chain management is the more comprehensive set of tasks involved in sourcing, manufacturing, and shipping goods to clients. Both are essential for companies to run smoothly and effectively satisfy client demand. Inventory management decisions and optimal inventory levels are achieved through the application of quantitative methodologies known as mathematical inventory models. These models often use mathematical formulae and algorithms to figure out the best ordering strategies, inventory levels, and reorder points.

An inventory cost-minimization methodology known as EOQ is used to identify the best order quantity. The order-placing cost and carrying cost are balanced in the model. Different inventory control strategies specify the amount and timing of orders. Common policies include placing orders at predetermined intervals, reviewing inventory levels periodically to determine order quantities, placing orders whenever inventory levels reach a predetermined reorder point, and placing orders to replenish inventory whenever it drops below a predetermined base stock level. In general, an inventory supply chain policy is developed win- win outcomes for both buyer and vendor. In this motive we develop inventory control supply chain policy with varying unit price and transportation cost.

1.1. **Research gap and our contribution.** Every inventory model has particular presumptions and works best under particular circumstances. A number of variables, including lead times, production capacities, cost structure, and demand prediction, influence the choice of model. In many research papers taken ordering and holding expenses into account, the EOQ model calculates the ideal order quantity that minimizes the overall cost. In this paper we discussed about the variable x be the order to which the inventory is raised in the beginning of a run of interval, y be the shortage arises and it increases from zero to the interval of time and we have considered the varying transportation cost for transporting ordering goods from origin to destination.

The partial differential equations have been considered with respect to the quantity and varying inventory of goods. Here we have considered the total cost function as the sum of ordering cost, holding cost, shortage cost, varying transportation cost and unit variable cost per order. Businesses can lower inventory-related expenses like storage, handling, and stockouts by using mathematical models, which balance the costs of ordering, holding, and possible stockouts, resulting in cost-effective inventory management. In addition to providing data-driven insights that help managers make well-informed decisions about ordering quantities, frequency, and safety stock levels, mathematical inventory models also frequently incorporate forecasting techniques to predict future demand based on historical data, enabling businesses to better plan their inventory and react to demand fluctuations.

1.2. **Study of flow.** The focus of this paper is to address the retailer supply chain system where the Inventory flow analysis is frequently done using mathematical model Economic Order Quantity (EOQ). Accurate demand forecasting is one of the most difficult aspects of inventory management. Seasonal variations, market trends, or outside events can all cause demand to fluctuate erratically.A number of factors, including production delays, shipping problems, customs clearance, and supplier performance, can affect the lead time. Overstocking raises holding costs and the risk of obsolete inventory, while stockouts result in lost sales, disgruntled customers, and the need for emergency replenishments. It is difficult to balance inventory levels to prevent both overstocking and stockouts, which lead to higher holding costs and decreased profitability. A system's flow of commodities, including how quickly inventory is used up, refilled, and stored in stock, is captured by these models. In this paper we have considered the variable x be the order to which the inventory is raised in the beginning of a run of time interval t and y be the shortage arise from zero during the interval of time.

2. Literature review under keywords

Sarkar, B., & Sarkar,S [1] discussed about the inventory model for degrading items, which sets the sufficient and necessary criteria for the existence and uniqueness of the optimal solution, and which is reliant on inventory levels and changes the pace of deterioration in some circumstances. To find the best ordering amount and replenishment cycle time so that the total profit per unit time is maximized, a straightforward algorithm has been suggested. Sarkar, B.[2] In an imperfect production process affected by inflation, the study examines an economic manufacturing quantity (EMQ) model incorporating price and advertising demand patterns. To make the defective things like new, they must be reworked at a cost. With time, more things of lower quality are produced. The author took into consideration that the cost of development, production, and materials is reliant on the dependability parameter in order to limit the creation of faulty things. Taking reliability into account as a variable for decision-making, the author creates an integrated profit function that control theory maximizes. Goyal, S. K., & Giri, B. C[3] reviewed the analysis of declining inventory, including fixed-life perishable inventory and inventory subject to ongoing exponential decay, and categorized the inventory based on changes in demand as well as other variables or limitations.

Najafnejhad, E et al [4] studied about a mathematical inventory model for a single-vendor multi-retailer supply chain which analysed on the Vendor Management Inventory Policy. Ebrahimi, S. B et al [5] performed problem consists of two objective functions, the problem as a single objective to obtain the lower and upper bounds value of the goals. Mishra, U et al [6] studied how greenhouse managers should invest in safeguarding and green technologies and introduce trade credit to increase their profits. Alkahtani, M et al [7] discussed about the mathematical model and optimizing the process outsourcing of damaged product with variable quantity for the effective supply chain management. Nasrollahi, M et al [8] studied about the combined Medicinal Supply Chain design with an expected coverage in the different hospitals with specific reliability value for different pharmaceutical substances. Mohammadi, H et al [9] discussed about worsening and seasonal products, such as fresh product supply and disposal of the worsened products are presented as a mathematical model of the location-routing problem.Davizon, Y. A et al [10] studied the mathematical modelling, optimal control, and stability analysis for dynamic supply chain. Aghsami, A et al [11] addresses various aspects of the blood collection centres and blood storage and its deteriorations are considered over a planning horizon. Yadav, A. S et al [12] discussed the possible level of excess stock and shortage required for green supply chain stocks of industry to find the minimum total cost in supply chain. Jagadeesan, V et al [13] established a production, re-do, and idle slots are the three distinct states of the production unit in the two-warehouse production inventory.

The production slot produces new things, the re-do slot re-does detected defective items, and the idle slot is in an idle state. The optimal total production cycle cost is determined explicitly in relation to the total production slot time and total cycle duration of the optimization variables. It was accomplished in each warehouse with a common demand rate but with varying rates of deterioration. Using the discriminant method, an analytical expression for the total cost of the production cycle is determined and optimized. Cortés, P et al [14] has presented a new mixed-integer linear programming model that builds on the foundations of production-inventory models and is inspired by aggregate production planning principles. Its goal is to optimize the network of distributed energy resources. The energy demands for heating and electricity over multiple periods can be satisfied by existing energy storage nodes or by a set of energy generation nodes.Jauhari et al [15]This study examines a mathematical inventory model for a supply chain with closed loops with just one manufacturer and one retailer in the setting of imperfect manufacturing and a stochastic environment. The manufacturer returns returned goods from the market by implementing a remanufacturing policy.

Daset al [16]discussed how a green product's price affects consumer demand and purchasing patterns, how the best inventory management techniques are adapted to various payment methods, including credit, cash, and advance payments, and how pricing strategy is most effective in striking a balance between market demand and profitability.Limi, A. et al [17]introduces a new sustainable inventory model that takes into account time-dependent holding costs and quadratic demand, specifically designed for non-instantaneous deteriorating items. Shortages are permitted because the holding cost has been deemed time-dependent.Farel [18] developed a cost description and came to the conclusion that the continuous review (Q) back order technique was selected because it had the lowest overall inventory costs. The company method and the Q back order method could be compared. [19] determined the ideal order quantity, prevented excess and shortage of stock, and more effectively reduced inventory costs by analyzing the integrated circuit raw material inventory using the Just in Time and Economic Order Quantity methods. Safety stock analysis, reorder point, and total inventory cost are all included to provide a more complete picture of optimal inventory management.

3. Problem discription, Assumptions & Notations

This inventory mathematical model's goal is to limit the overall expenses incurred by the retailer, buyer, or vendor that buys the final goods from the manufacturer or vendor and resells them to consumers. We talked about the store sending goods to customers via shipping, which incurs transportation expenses, in this paper. The merchant will be responsible for overseeing the overall expenses, which include setup, shortfall, unit variable, and shipping costs. The assumptions and symbols used in this model are as follows:
D- Demand per unit per item
A- set up cost (in \$)per cycle.
Shortages are allowed.
Q- Order quantity per item per order.
h- holding cost (in \$) per unit per time period
s- Shortage cost (in \$)per unit per time period
x- be the order to which the inventory is raised in the beginning of a run of time interval t.

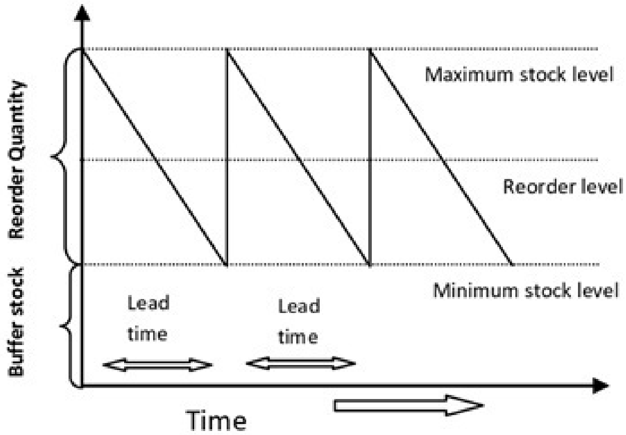

FIGURE 1. Inventory Mathematical Model's

y- be the shortage arise and increase from zero to the remaining interval of time.
B_0 - Transportation cost (in \$) per order per quantity.
TAC- Total Annual Cost (in \$)
Q*- Optimum order quantity per order.

4. Model framework of supply chain

The mathematical representation of players 1 and 2 serves as a buyer and retailer in this model. This model is suitable for the retailer than the vendor. Mathematically, the retailer's total cost function is defined as follows.

Total cost = Ordering cost + holding cost + shortage cost + varying transportation cost + Unit variable cost per order.

$$TC = \frac{DA}{Q} + \frac{1}{2}\frac{hx^2}{Q} + \frac{1}{2}\frac{y^2 s}{Q} + B_0 Q + IQ \qquad (1)$$

where$y = Q - x$

$$TC(Q,x) = \frac{DA}{Q} + \frac{1}{2}\frac{hx^2}{Q} + \frac{1}{2}\frac{(Q-x)^2 s}{Q} + B_0 Q + IQ$$

$$\frac{\partial TC}{\partial Q} = DA\left(\frac{-1}{Q^2}\right) + \frac{hx^2}{2}\left(\frac{-1}{Q^2}\right) + \frac{S}{2}\left(1 + \frac{x^2}{-Q^2}\right) + B_0 + I$$

$$\frac{\partial TC}{\partial Q} = \frac{-DA}{Q^2} - \frac{hx^2}{2Q^2} + \frac{s}{2} - \frac{sx^2}{2Q^2} + B_0 + I \qquad (2)$$

$$\frac{\partial TC}{\partial Q} = 0$$

$$\begin{aligned}
\Rightarrow -\frac{DA}{Q^2} - \frac{hx^2}{2Q^2} - \frac{sx^2}{2Q^2} &= -(B_0 + I + \frac{s}{2}) \\
\frac{1}{Q^2}\left(DA + \frac{hx^2}{2} + \frac{sx^2}{2}\right) &= B_0 + I + \frac{s}{2} \\
\frac{2DA + hx^2 + sx^2}{2} &= Q^2\left[\frac{2B_0 + 2I + s}{2}\right] \\
Q &= \sqrt{\frac{2DA + (h+s)\frac{s^2Q^2}{(h+s)^2}}{2(B_0+I)+s}} \\
Q^2 &= \frac{2DA + \frac{s^2Q^2}{h+s}}{2(B_0+I)+s} \\
Q^2 &= \frac{2DA}{2(B_0+I)+s} + \frac{s^2Q^2}{(h+s)[2(B_0+I)+s]} \\
Q^2\left[1 - \frac{s^2}{(h+s)[2(B_0+I)+s]}\right] &= \frac{2DA}{2(B_0+I)+s} \\
Q^2\left[\frac{(h+s)[2(B_0+I)+s] - s^2}{(h+s)[2(B_0+I)+s]}\right] &= \frac{2DA}{2(B_0+I)+s} \\
Q^2 &= \frac{2DA(h+s)[2(B_0+I)+s]}{(h+s)[2(B_0+I)+s] - s^2} \\
Q^2 &= \frac{2DA(h+s)}{(h+s)[2(B_0+I)+s] - s^2} \\
&= \frac{2DA(h+s)}{h[2(B_0+I)+s] + 2s(B_0+I) + s^2 - s^2} \\
&= \frac{2DA(h+s)}{(h+s)[2(B_0+I)] + sh} \qquad (3) \\
\frac{\partial TC}{\partial x} &= \frac{h}{2Q}(2x) + \frac{2s}{2Q}(Q-x)(-1) \qquad (4) \\
\frac{hx}{Q} - \frac{s(Q-x)}{Q} &= 0 \\
x &= \frac{sQ}{h+s} \qquad (5)
\end{aligned}$$

The ordering Quantity is

$$\begin{aligned}
Q &= \sqrt{\frac{2DA(h+s)}{2(B_0+I)(h+s)+sh}} \qquad (6) \\
\frac{\partial^2 TC}{\partial Q^2} &= -DA\left(\frac{-2Q}{Q^4}\right) - \frac{hx^2}{2}\left(\frac{-2Q}{Q^4}\right) - \frac{sx^2}{2}\left(\frac{-1}{Q^4}(2Q)\right)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 TC}{\partial Q^2} &= \frac{1}{Q^3}\left[\frac{2DA(h+s)+s^2Q^2}{h+s}\right] && (7)\\
\frac{\partial^2 TC}{\partial Q^2} &= \frac{2h}{2Q}+\frac{2s}{2Q}(0-1)\\
\frac{\partial^2 TC}{\partial Q^2} &= \frac{h+s}{Q} && (8)\\
\frac{\partial^2 TC}{\partial Q \partial x} &= xh\left(\frac{-1}{Q^2}\right)-s\left(0+\frac{x}{Q^2}\right)\\
&= -\frac{x(s+h)}{Q^2} && (9)\\
\frac{\partial^2 TC}{\partial Q^2}\frac{\partial^2 TC}{\partial x^2}-\frac{\partial^2 TC}{\partial Q \partial x} &= \frac{[2(B_0+I)(h+s)+hs][2(B_0+I)+s]}{2DA}+s\\
&= \frac{[2(B_0+I)(h+s)+hs][2(B_0+I)+s]}{2DA}+s \geq 0 && (10)
\end{aligned}$$

$$\text{For optimality,}\frac{\partial^2 TC}{\partial Q^2} \geq 0 \ \text{ and } \frac{\partial^2 TC}{\partial Q^2}\frac{\partial^2 TC}{\partial x^2}-\left[\frac{\partial^2 TC}{\partial Q \partial x}\right]^2 \geq 0$$

$$\begin{aligned}
\frac{\partial^2 TC}{\partial Q^2} &= \frac{2DA(h+s)}{Q^3(h+s)}+\frac{s^2Q^2}{Q^3(h+s)}\left[\frac{\partial^2 TC}{\partial Q^2}\right]_{Q=\sqrt{\frac{2DA(h+s)}{2(B_0+I)(h+s)+sh}}}\\
&= [2(B_0+I)+s]\sqrt{\frac{2(B_0+I)(h+s)+sh}{2DA(h+s)}} \geq 0\\
\frac{\partial^2 TC}{\partial x^2} &= \frac{h+s}{Q} && (11)\\
\frac{\partial^2 TC}{\partial x^2} &= \frac{h+s}{\sqrt{\frac{2DA(h+s)}{2(B_0+I)(h+s)+sh}}}\\
\frac{\partial^2 TC}{\partial x^2} &= \frac{(h+s)\sqrt{2(B_0+I)(h+s)+sh}}{\sqrt{2DA(h+s)}}\\
\left[\frac{\partial^2 TC}{\partial x^2}\right]_{\sqrt{\frac{2DA(h+s)}{2(B_0+I)(h+s)+sh}}} &= \frac{(h+s)\sqrt{2(B_0+I)(h+s)+sh}}{\sqrt{2DA(h+s)}}
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{\partial^2 TC}{\partial Q^2} &\geq 0,\\
\frac{\partial^2 TC}{\partial x^2} &\geq 0\\
\frac{\partial^2 TC}{\partial Q \partial x} &= \frac{-x(h+s)}{Q^2}\\
\frac{\partial^2 TC}{\partial Q \partial x} &= -s < 0
\end{aligned}$$

Since the shortage cost is very small and it will not affect the condition minimum. Also, the value of the second derivative test is positive which is greater than zero. Using second derivative test of Partial differential equation $rs - t^2 > 0$ (where r, s and t are partial derivatives) implies the function is minimum. Hence by the necessary and sufficient condition for maxima and minima we get $\frac{\partial^2 TC}{\partial Q^2}\frac{\partial^2 TC}{\partial x^2} - \left[\frac{\partial^2 TC}{\partial Q \partial x}\right]^2 > 0$ is satisfied. We assumed that the order quantity which is greater than 1. So there won't be any restriction for the denominator being zero.Some inventory models assume that certain variables such as lead time, demand, or reorder quantity must remain within certain bounds if a variable fall outside those bounds, the function may become undefined. Particularly, those models that rely on assumptions about demand may exhibit undefined behaviour if the underlying assumptions are violated.

The Total variable cost is the optimal (minimum) Inventory cost.

Therefore,

$$TC = \frac{DA}{Q} + \frac{1}{2}\frac{hx^2}{Q} + \frac{1}{2}\frac{y^2 s}{Q} + B_0 Q + IQ$$

The total variable cost

$$\begin{aligned} TC^* &= \frac{2DA}{\sqrt{\frac{2DA(h+s)}{2(B_0+I)(h+s)+sh}}} \\ TVC^* &= \sqrt{2DA\big[2(B_0+I)(h+s) + \frac{sh}{s+h}\big]} \end{aligned}$$

Hence the optimum order quantity $Q^* \sqrt{\frac{2DA(h+s)}{2(B_0+I)(h+s)+sh}}$ and the minimum inventory total cost is

$$TC^*(Q^*) = \sqrt{2DA\big[2(B_0+I)(h+s) + \frac{sh}{s+h}\big]}$$

5. SOLUTION METHODOLOGY

In this paper we optimize the total cost by using the method of differential calculus and optimum solution has got from MATLAB. If the demand rate is continuous and deterministic, the goal is to determine when and how much to order can be modelled by solving the differential equation, optimizing the decision over time, Optimize ordering policies, Determine optimal inventory levels over time in continuous inventory systems, Minimize total cost functions involving ordering, holding, and shortage costs. The primary advantage of using differential calculus is that it provides continuous optimization, especially in continuous-time models, making it easier to model and optimize inventory.

6. NUMERICAL ANALYSIS

The following table analyses the optimum quantity for running of smooth business and total annual cost of the retailer based on the set-up cost, transportation cost, item cost, holding cost and shortage cost using MATLAB. The table shows the optimum cost for the business.The values are taken at random which gives the optimum values by using MATLAB.

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D (units)	A(in \$)	B0(in \$)	I(in \$)	H(in \$)	S(in \$)	Q*	TAC Z(in \$)
1000	3000	3	20	0.75	0.05	361	16622
1000	3000	3	20	0.8	0.05	361	16622
1000	3000	3	20	0.85	0.05	361	16622
1000	3000	3	20	0.9	0.05	361	16622
1000	3000	3	20	0.95	0.05	361	16622
1000	3000	3.1	20	0.85	0.05	360	16658
1000	3000	3.2	20	0.85	0.05	359	16694
1000	3000	3.3	20	0.85	0.05	359	16730
1000	3000	3.4	20	0.85	0.05	358	16766
1000	3000	3.5	20	0.85	0.05	357	16801
1000	3000	3	21	0.85	0.05	349	17190
1000	3000	3	22	0.85	0.05	342	17535
1000	3000	3	23	0.85	0.05	336	17874
1000	3000	3	24	0.85	0.05	330	18207
1000	3000	3	25	0.85	0.05	324	18533
1100	3000	3	20	0.85	0.05	379	17433
1200	3000	3	20	0.85	0.05	395	18208
1300	3000	3	20	0.85	0.05	412	18952
1400	3000	3	20	0.85	0.05	427	19667
1500	3000	3	20	0.85	0.05	442	20357
1000	3100	3	20	0.85	0.05	367	16897
1000	3200	3	20	0.85	0.05	373	17167
1000	3300	3	20	0.85	0.05	379	17433
1000	3400	3	20	0.85	0.05	384	17695
1000	3500	3	20	0.85	0.05	390	17954
1000	3000	3	20	0.85	0.1	361	16629
1000	3000	3	20	0.85	0.2	361	16642
1000	3000	3	20	0.85	0.3	360	16653
1000	3000	3	20	0.85	0.4	360	16662
1000	3000	3	20	0.85	0.5	360	16670

Table 1. Various costs involved in this model

7. Managerial Insights

EOQ formula provides an optimal order quantity that minimizes total inventory costs, which include ordering and holding costs. Managers can use this EOQ model

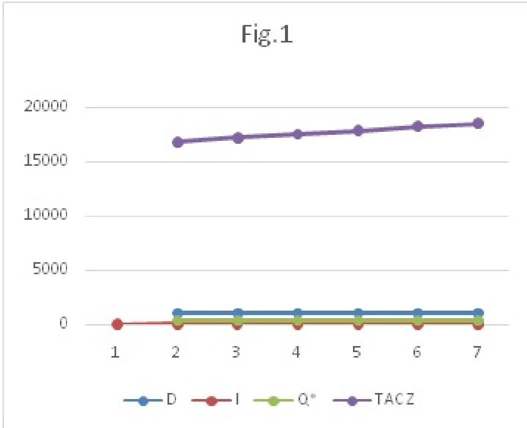

FIGURE 2. Effect of changes when unitcost changes

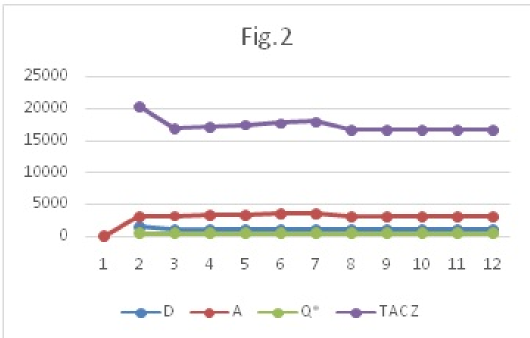

FIGURE 3. Effect of changes when setup cost changes

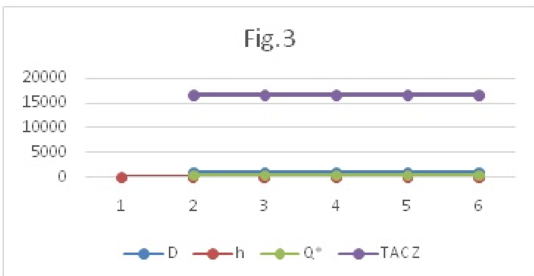

FIGURE 4. Effect of changes when holding cost changes

to determine the most cost-efficient quantity to order each time, to predict demand and modify inventory control measures as necessary. They can lower the risk of stock outs or excess inventory by integrating variability into inventory decisions,

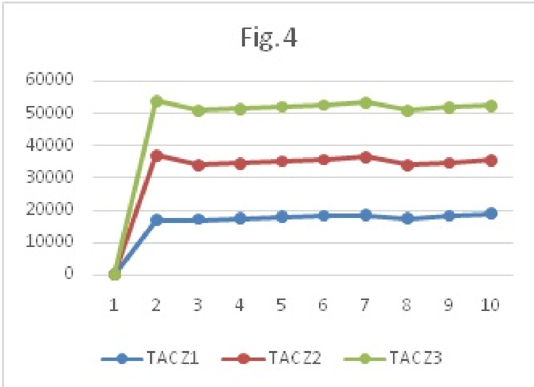

Figure 5. comparison of TAC Z with various system

which will better prepare them for variations in demand. Based on the fluctuations in lead time and demand, managers must establish the proper safety stock levels. While too little safety stock can lead to stock outs, too much safety stock ties up cash. This helps in avoiding overstocking or under stocking and to determine when to reorder products and how much to order.

8. Conclusion and Future Extension

In this paper we developed a mathematical model for the situation where we find supply chain problem to find the optimal quantity of items to be supplied based on the demand. Here we analyzed various costs which are involved in this model and obtained EOQ for running the business effectively and the total annual inventory costs when some changes in the parameters. Fig.1 represents the effect of changes when the item costs vary and the demand, setup cost, holding costs, transportation costs remain unchanged. In this condition we could identify that the total annual inventory cost also increases gradually. Fig.2 represents the effect of changes when setup cost varies and the other parameters remain unchanged. In this condition we could find the minor fluctuation occurs in the total annual inventory cost. Fig.3 represents the effect of changes when the holding cost changes while the other parameters remain unchanged. That is based on the minor fluctuation of holding cost the total inventory costs shows that it is linear because of minor fluctuation in the holding cost.

If there is big increase in the holding cost then it will show the great increase in the total inventory cost which will affect the smooth running of supply chain. Also, we have taken the slight variation in the transportation cost which will not affect much in the total inventory cost as the goods will be supplied as much as possible. Fig.4 represents the comparison of the three conditions what happens to the total annual inventory cost when the other parameters have fluctuations. We conclude this paper by finding the total annual inventory cost increases when there is an increase in the holding cost, transportation cost, setup cost and item cost. So, the business management can make decision about the inventory policy which minimizes the total yearly inventory cost for the effective supply chain and for the optimum profit in the business.This paper's extension could address different demand patterns, manufacturing models, holding costs, and transportation costs, both with and without buyer-vendor coordination, in order to determine the best overall cost and profit for the manufacturer, buyer, vendor or retailer.

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